Ableiten mit Produkt- und Kettenregel

a)

$$f(x) = \frac{x^2+6x}{(x-4)^2} = \frac{x^2+6x}{1} \cdot \frac{1}{(x-4)^2}$$

$$= (x^2+6x) \cdot [x-4]^{-2}$$

$$f'(x) = (2x+6)\cdot[x-4]^{-2} + (x^2+6x)\cdot(-2)\cdot[x-4]^{-3}\cdot 1$$

$$= \frac{2x+6}{(x-4)^2} + \frac{-2(x^2+6x)}{(x-4)^3}$$

$$= \frac{(2x+6)(x-4)}{(x-4)^2(x-4)} + \frac{-2(x^2+6x)}{(x-4)^3}$$

$$= \frac{(2x+6)(x-4)}{(x-4)^3} + \frac{-2(x^2+6x)}{(x-4)^3}$$

$$= \frac{2x^2-8x+6x-24}{(x-4)^3} + \frac{-2x^2-12x}{(x-4)^3}$$

$$= \frac{2x^2-2x-24}{(x-4)^3} + \frac{(-2x^2-12x)}{(x-4)^3}$$

$$= \frac{2x^2-2x-24+(-2x^2-12x)}{(x-4)^3}$$

$$= \frac{\cancel{2x^2}-2x-24-\cancel{2x^2}-12x}{(x-4)^3}$$

$$= \frac{-14x-24}{(x-4)^3}$$

b)

$$f(x) = \frac{(x^2-3)^2}{\cos(x)} = \frac{(x^2-3)^2}{1} \cdot \frac{1}{\cos(x)}$$

$$= (x^2-3)^2 \cdot [\cos(x)]^{-1}$$

$$f'(x) = 2(x^2-3)^1 \cdot 2x \cdot [\cos(x)]^{-1} + (x^2-3)^2 \cdot (-1) \cdot [\cos(x)]^{-2} \cdot (-\sin(x))$$

$$= \frac{4x\cdot(x^2-3)\cdot[\cos(x)]}{\cos(x)\cdot\cos(x)} + \frac{(x^2-3)^2(\sin(x))}{[\cos(x)]^2}$$

$$= \frac{4x\cdot(x^2-3)\cdot[\cos(x)]}{[\cos(x)]^2} + \frac{(x^2-3)^2(\sin(x))}{[\cos(x)]^2}$$

$$= \frac{(4x^3-12x)\cdot[\cos(x)]}{[\cos(x)]^2} + \frac{(x^2-3)^2(\sin(x))}{[\cos(x)]^2}$$

$$= \frac{(4x^3-12x)\cdot[\cos(x)] + (x^2-3)^2\cdot(\sin(x))}{\cos^2(x)}$$